Structural Model Validation and the Lack-Of-Knowledge Theory

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Today, the validation of complex structural models - *i.e.* the assessment of their quality compared to an experimental reference - remains a major issue. The true validation problem should be addressed through the comparison between the model - whether deterministic or not - used classically and the complete reality: such an issue raises philosophical questions [1].

Here, we introduce a tentative answer through the Lack-Of-Knowledge (LOK) theory [2, 3], whose aim is to ”model the unknown”. Of course, the theory takes into account all the sources of uncertainties, including modeling errors, through the concept of basic LOKs: a set of basic LOKs is added to the classical model to constitute the true model. This leads to an envelope of the actual responses; in particular, we can derive for the whole structure the effective LOK of a quantity of interest α, resulting in an interval with stochastic bounds.

This paper focuses on the basic ideas of the Lack-Of-Knowledge theory and on its first applications. Academic examples as well as industrial cases are presented.

1 Introduction

Model validation is becoming a rather hot topic, even if some people tend to say that a model can only be invalidated, but never validated. Much has been written on this controversial issue (see [4, 5, 6, 7, 8]). On the positive side, the word “validation” is used very often by engineers in the sense of a procedure, defined through practical rules and experiments, which ensures that the structure being designed will, once built, perform as expected. Where there is a lack of knowledge, safety coefficients are introduced in order to guarantee conservative predictions. Thus, model validation in the context of "engineering reality" should be considered a real issue. The challenge is to go beyond the philosophical level and elaborate practical tools which can be applied to true engineering problems.

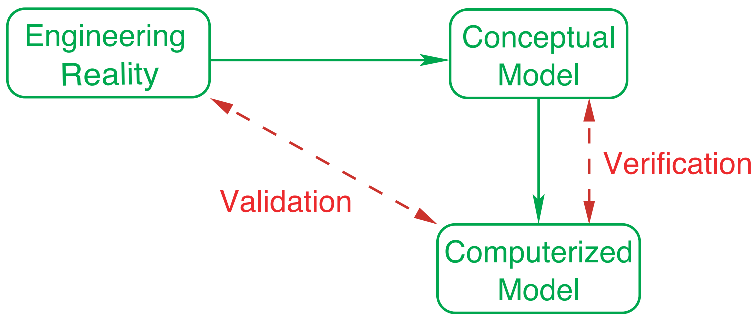

Fig. 1. Model validation and verification

Verification is generally considered to be a related topic, which can be viewed as a subset of model validation using an intermediate, but perfectly defined, reference: the conceptual model. Thus, verification is a rather well-posed problem and an easier challenge (see Fig. 1).

In this paper, we first review the current state-of-the-art [9, 10] and deal with a model's validation with respect to a particular set of experimental data and with the updating of the model which it leads to. Then, we investigate the actual validation problem. We develop a tentative answer through the Lack-Of-Knowledge (LOK) theory, whose aim is to "model the unknown" in a conservative way. This theory was introduced in [2] and its first application was presented in [3].

The LOK theory can be interpreted as an extension of what design engineers do when they introduce safety factors. Of course, the theory takes into account all the sources of uncertainties, including modeling errors, through the concept of basic LOKs. So far, two kinds of basic LOKs have been introduced: stiffness and excitation. For example, considering the structure as an assembly of substructures, the basic stiffness LOKs are defined on the substructure level: each LOK is a pair of scalar internal variables which define upper and lower bounds of the real substructure's stiffness. In mathematical terms, these bounds can be deterministic, in which case they can be viewed in a certain way as safety coefficients; more generally, they can follow probabilistic laws.

Finally, a set of basic LOKs is added to the classical model to constitute the true model. This leads to an envelope of the actual responses; in particular, we can derive for the whole structure the effective LOK of a quantity of interest α, resulting in an interval with stochastic bounds. Another major question is the reduction of the LOKs using additional experimental information; the starting point could be an overestimated initial LOK level coming from experience.

The paper focuses on the basic ideas of the Lack-Of-Knowledge theory and on its first applications. Academic examples as well as industrial cases will be presented.

2 Validation with respect to a particular set of experimental data

A first, restrictive problem is model validation with respect to to a well-chosen (but particular) set of experimental data, and the resulting updating of the model. Much work has been done in this area since 1980 and, today, there are some engineering tools available; the subject which seems to have seen the most progress is the updating of dynamic structural models (mass, stiffness, damping) in the low-frequency range (see [9, 10]). Many of the methods proposed do not attempt to provide meaningful error measures which could be used for validation: their only objective is updating. A first set of methods is based on the search for minimum norm corrections. A second set is closely related to control theory. There are two main difficulties. First, the updating problem, like all inverse problems, is not well-posed and requires regularization. The second difficulty concerns the localization of the corrections: sensitivity techniques or the like are required to identify the most erroneous structural parameters. All these methods work well when a sufficiently large amount of experimental data is available, which is not always the case. Let us also note that other difficulties could come from the measurements; an important practical issue is how to eliminate erroneous measurements due to human error. True error indicators in the framework of the Constitutive Relation Error (CRE) method were developed in [10, 11, 12] in an attempt to overcome these difficulties. At the core of this approach is the question of the choice of the reference. In the CRE approach, the reference contains the ”reliable” part of the model, e.g. the equilibrium equations. Only a subset of the experimental data - the reliable data - is part of the reference. Actually, two errors are calculated: the modeling error, and an error which characterizes the quality of the experimental data. The capability of this approach is illustrated below.

Figure 2 shows the structure being studied, a satellite support called SYLDA5, for which a FE model is available. A preliminary step, called ”recovery of the experimental results” was first performed using the error characterizing the quality of the experimental data. Erroneous sensors were localized, then corrected (if possible) or removed. Next, the updating process, which is iterative, began. At each iteration, the most erroneous structural parameters were localized and corrected until the modeling error reached a threshold of a few percent. At the starting point, the modeling error was 12.39 %, which is quite significant; the updating process was carried out until the modeling error became reasonable, i.e. 2 or 3 %. After four iterations, the error was down to 2.27 %. Table 1 lists separately the ”total” modeling error (E_{CRE_T}), the modeling error of each mode (E_{CRE}), and the differences between the calculated and measured frequencies at each iteration. A much better fit can be observed after 4 iterations. One can also note that the calculated CRE-modeling errors on the

Fig. 2. The structure

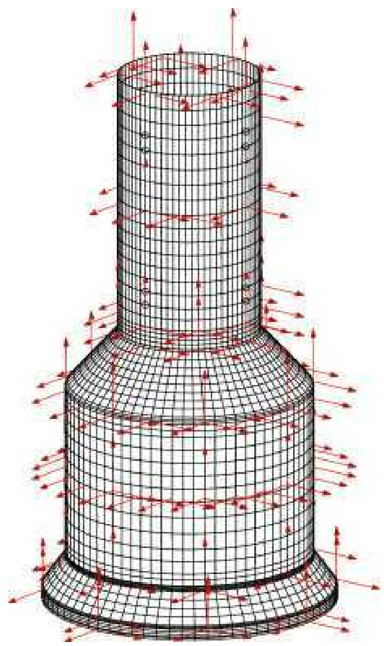

Fig. 3. The experimental data

modes are nearly identical to the errors on the frequencies, which are the differences Δf between the calculated and measured values.

Table 1. Summary of the updating process. ($^+$: mode swapped with Modes 4 and 5)

Iterations	Initial		1		2		3		4	
Error (%)	E_{CRE}	Δf	E_{CRE}	Δf	E_{CRE}	Δf	E_{CRE}	Δf	E_{CRE}	Δf
Mode 1	3.45	-3.51	2.65	-2.69	0.24	0.28	2.05	2.05	1.67	1.86
Mode 2	3.25	-3.31	2.46	-2.49	0.39	0.37	2.19	2.15	1.54	1.43
Mode 3	5.10	-5.24	4.06	-4.14	0.40	-0.40	1.49	1.48	1.49	1.48
Mode 4	10.29	-10.85	9.78	-10.29	9.03	-9.48	4.09	-4.15	4.28	-4.22
Mode 5	9.86	-10.39	9.36	-9.83	8.61	-8.99	3.66	-3.75	3.76	-3.98
Mode 6	3.47	3.45^+	1.91	1.90^+	0.10	0.10^+	2.66	2.63	0.00	0.00
Mode 7	27.15	23.60	1.27	1.27	1.27	1.27	3.36	3.30	2.71	2.67
Mode 8	27.77	24.09	1.92	1.90	1.92	1.90	4.00	3.92	3.36	3.30
Mode 9	1.78	-1.79	0.09	-0.09	0.09	-0.09	0.02	0.02	0.10	-0.10
Mode 10	1.24	-1.24	0.44	0.44	0.44	0.44	0.55	0.55	0.43	0.43
Mode 11	6.41	*	1.01	*	1.04	*	2.11	*	1.65	*
Mode 12	4.82	*	0.64	*	0.63	*	1.36	*	0.56	*
(E_{CRE_T})	12.39		4.32		3.69		2.62		2.27	

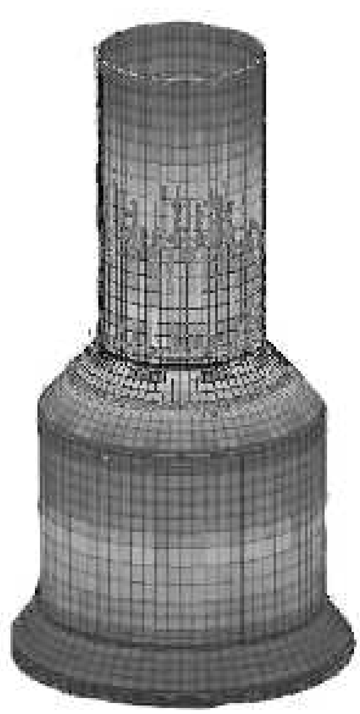

Fig. 4. Map of the local modeling error

<u>Remarks:</u> A possible situation is that the modeling error does not decrease enough to reach the target; this means that the model is too coarse.

An additional development to include probabilistic models and noisy experimental data was presented in [13].

3 Model validation: the real problem

The description of parameters such as material uncertainties is necessarily at the heart of any approach to the actual validation problem in the context of engineering reality. Probabilistic modeling has become increasingly popular [14]. There are also other approaches which do not involve probability laws, such as in [15, 16, 17, 18].

Here, we propose an approach, called the Lack-Of-Knowledge theory (LOK), which attempts to give a pragmatic answer to the problem of model validation in the context of engineering reality. This could be viewed as an extension of the concept of safety coefficients [2, 3].

3.1 The concept of basic LOKs

For the sake of simplicity, let us consider a family of quasi-identical real structures subjected to a given (but not very well-known) time-independent environment.

3.1.1 Basic LOKs on the stiffness

The structure is modeled as an assembly of substructures E ($E \in \mathbb{E}$) whose connections can be viewed as special substructures. The corresponding scale is assumed to be consistent with the outputs of interest being sought. We also assume that an elastic, deterministic FE model is sufficient to predict the response of the "structure" in the usual sense.

The starting point of the LOK theory consists in associating with each substructure E a pair of scalar internal state variables $(m_{\mathrm{E}}^-, m_{\mathrm{E}}^+)$, called the "basic LOKs", such that:

$$-m_{\mathrm{E}}^-(\theta)\overline{\mathbb{K}}_{\mathrm{E}} \leq \mathbb{K}_{\mathrm{E}}(\theta) - \overline{\mathbb{K}}_{\mathrm{E}} \leq m_{\mathrm{E}}^+(\theta)\overline{\mathbb{K}}_{\mathrm{E}} \tag{1}$$

where $\overline{\mathbb{K}}_{\mathrm{E}}$ is the calculated stiffness matrix and $\mathbb{K}_{\mathrm{E}}(\theta)$ the stiffness matrix of an actual structure belonging to the family being studied. In this formal expression, the inequalities must be considered to hold for the eigenvalues. The basic LOKs (m_{E}^+ and m_{E}^-) could take set values, but they usually follow given probability laws. Let us note that a more simple description would be to replace the interval $[-m_{\mathrm{E}}^-(\theta); m_{\mathrm{E}}^+(\theta)]$ by a fuzzy interval depending on only one stochastic variable.

3.1.2 Basic LOKs on the applied forces

The definition of the basic LOKs on the applied forces (and, more generally, on the description of the environment) is similar to that of the basic LOKs on the structural stiffness. Let us consider a force distribution $\underline{F}$ which can be written as:

$$\underline{F} = \sum_{E \in \mathbb{E}} \underline{F}_E \tag{2}$$

where $\underline{F}_E$ is the restriction to Substructure E of $\underline{F}$ at the boundary of E or over the E-domain. Furthermore, let us consider that:

$$\underline{F}_E = \lambda_E \underline{Z}_E \tag{3}$$

with $\overline{e}_E(\underline{Z}_E) = 1$. $\overline{e}_E$ is the E-energy obtained using the finite element model restricted to E. λ_E is the amplitude and $\underline{Z}_E$ the direction of the load restriction $\underline{F}_E$. In this case, the associated basic LOK is a triplet for each $E \in \mathbb{E}$, which is generally stochastic, but could have set values. This LOK is defined by:

$$-m^-_{\lambda_E}(\theta)\overline{\lambda}_E \leq \lambda_E(\theta) - \overline{\lambda}_E \leq m^+_{\lambda_E}(\theta)\overline{\lambda}_E \quad \forall E \in \mathbb{E} \tag{4}$$

$$[\overline{e}_E(\underline{Z}_E(\theta) - \overline{\underline{Z}}_E)]^{\frac{1}{2}} \leq m_{Z_E}[\overline{e}_E(\overline{\underline{Z}}_E)]^{\frac{1}{2}} \quad \forall E \in \mathbb{E} \tag{5}$$

Here, the model of the environment being used is considered to be deterministic. $\underline{F}(\theta)$ followed by $(\lambda_E(\theta), \underline{Z}_E(\theta))_{E\in\mathbb{E}}$ is the actual applied force distribution, while $\overline{\underline{F}} = \sum(\overline{\lambda}_E \overline{\underline{Z}})$ is the calculated applied force distribution.

Remarks: More complex LOK descriptions can be introduced. For example, an anisotropic description could be used for highly anisotropic substructures.

3.1.3 Effective LOK on an output of interest

Let us first introduce the solution $\underline{x}$ of the equation:

$$\mathbb{K}\underline{x} = \underline{f} \tag{6}$$

where $\mathbb{K}$ is the stiffness matrix and $\underline{f}$ is the generalized force. Let $\underline{m}$ be the set of all the basic LOKs associated with the stiffness terms and the applied forces. It is clear that one can associate with each value of $\underline{m}$ the stiffness and the force which verify Equations (1), (4) and (5). The corresponding sets are $\mathcal{K}_m$ and $\mathcal{F}_m$, which are parameterized by $\underline{m}$.

This approach yields a new modeling approach for the family of actual structures being studied, in which one defines only the envelope of the actual responses. Precisely, the envelope of the actual responses is associated with Equation (6), where

$$\mathbb{K}(\theta) \in \mathcal{K}_{m(\theta)} \qquad \underline{F}(\theta) \in \mathcal{F}_{m(\theta)} \tag{7}$$

For an output of interest $\alpha(\theta)$ (whose FE value is $\overline{\alpha}$), one can calculate the effective LOK such that

$$-\Delta\alpha^-(\theta) \leq \alpha(\theta) - \overline{\alpha} \leq \Delta\alpha^+(\theta) \tag{8}$$

The output of interest α is a functional of $\mathbb{K}$ and $\underline{f}$:

$$\alpha = \mathbb{L}(\mathbb{K}, \underline{f}) \tag{9}$$

where $\mathbb{L}$ is a given operator. The FE value is:

$$\overline{\alpha} = \mathbb{L}(\overline{\mathbb{K}}, \overline{\underline{f}}) \tag{10}$$

Consequently, one has:

$$\Delta\alpha^+(\theta) = \sup_{\substack{\mathbb{K}(\theta)\in\mathcal{K}_{m(\theta)} \\ \underline{F}(\theta)\in\mathcal{F}_{m(\theta)}}} [\mathbb{L}(\mathbb{K}, \underline{f}) - \overline{\alpha}] \tag{11a}$$

$$\Delta\alpha^-(\theta) = -\inf_{\substack{\mathbb{K}(\theta)\in\mathcal{K}_{m(\theta)} \\ \underline{F}(\theta)\in\mathcal{F}_{m(\theta)}}} [\mathbb{L}(\mathbb{K}, \underline{f}) - \overline{\alpha}] \tag{11b}$$

If the basic LOKs are small enough, the calculation is easy using linearization. Let us consider:

$$\alpha = \underline{b}^T \underline{x} = (\mathbb{K}^{-1}\underline{b})^T \underline{f} \tag{12}$$

One has:

$$\Delta\alpha = (\mathbb{K}^{-1}\underline{b})^T \Delta\underline{f} - \overline{\underline{f}}^T \mathbb{K}^{-1}(\Delta\mathbb{K})\mathbb{K}^{-1}\underline{b} \tag{13}$$

and:

$$\begin{aligned}\Delta\alpha^+(\theta) = \sum_{E\in\mathbb{E}} [& m_E^+(\theta)\frac{1}{2}\overline{e}_E((\overline{\underline{x}} - \underline{x}_E)) + m_E^-(\theta)\frac{1}{2}\overline{e}_E((\overline{\underline{x}} + \underline{x}_E)) \\ & + m_{\lambda_E}^+(\theta)\underline{x}_{b_E}^T \overline{\underline{f}}_E + m_{Z_E}(\theta)[\overline{e}_E(\underline{x}_{b_E})]^{\frac{1}{2}}\overline{\lambda}_E] \end{aligned} \tag{14a}$$

$$\begin{aligned}-\Delta\alpha^-(\theta) = \sum_{E\in\mathbb{E}} [& -m_E^+(\theta)\frac{1}{2}\overline{e}_E((\overline{\underline{x}} + \underline{x}_E)) - m_E^-(\theta)\frac{1}{2}\overline{e}_E((\overline{\underline{x}} - \underline{x}_E)) \\ & - m_{\lambda_E}^-(\theta)\underline{x}_{b_E}^T \overline{\underline{f}}_E - m_{Z_E}(\theta)[\overline{e}_E(\underline{x}_{b_E})]^{\frac{1}{2}}\overline{\lambda}_E] \end{aligned} \tag{14b}$$

with

$$\underline{x}_b = \mathbb{K}^{-1}\underline{b} \tag{15}$$

3.1.4 Example: effective LOKs

Let us consider a hollow plate clamped on one side. A force distribution $\underline{F} = \lambda\underline{Z}$ (with, in this case, $\underline{Z} = \underline{y}$) is applied along the opposite side as shown in Figure 5. We are interested in the propagation of the LOK model described below.

Detail of the LOK model

Let us consider two LOKs $(m_\lambda^-, m_\lambda^+)$ on the amplitude of the force distribution (only one substructure being used for the applied force), and two LOKs (m_E^-,m_E^+) on the stiffness of each element (for the sake of simplicity, we consider that the basic LOKs ($m_{E_i}^-$,$m_{E_i}^+$) on the stiffness are the same for all elements.) We consider the direction of the force distribution to be well-known, whereas the LOKs on the amplitude

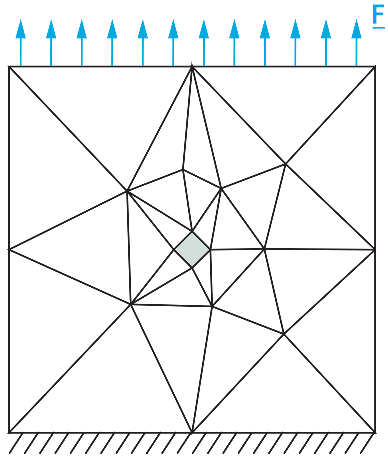

Fig. 5. Discretized model of the structure being studied

$(\overline{m}_F^-, \overline{m}_F^+)$ does not follow probability law (thus, the description of the LOK is similar to the description of an interval.) The LOKs on the stiffness coefficients of the elements of the plate follow a Gaussian probability law.

Our quantity of interest is the mean strain σ_{yy} in the finite element where it is maximum, i.e. Element 20 (see Figure 6). Therefore, $\alpha = \sigma_{yy}^{20}$.

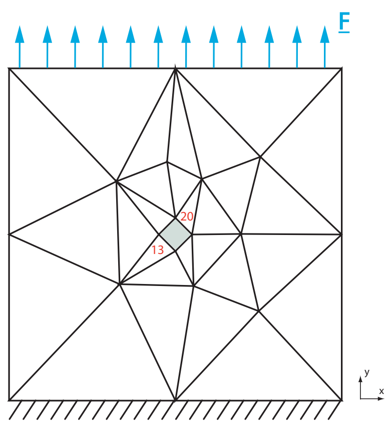

Fig. 6. Detailed view of the elements of the structure being studied

Since the values of the LOKs are assumed to be small enough, we use the linearized LOK propagation as described in Equation (11a). Thus, we can study the influence of each of the two dispersions separately: the values of the LOKs and the

99%-probability bounds of the quantity of interest obtained with the LOK model are shown in Tables 2 and 3. Table 2 presents the results of the LOK propagation when LOKs on the amplitude of the force distribution alone are considered; Table 3 presents the results of the whole LOK propagation (including LOKs on the amplitude and LOKs on the stiffness of each element).

Table 2. Results for an LOK model limited to a uniform force distribution dispersion

Range $[\overline{m}_F^-, -\overline{m}_F^+]$	Range $[-\overline{m}_{E_i}^-, -\overline{m}_{E_i}^+]$	Range [$-\Delta\sigma^{20\,-}_{\mathrm{yy\,mod}}$, $\Delta\sigma^{20\,+}_{\mathrm{yy\,mod}}$]
[-0,10 ; 0,10]	*	$[-1, 2.10^6; 1, 2.10^6] = \pm 9\%$ of $\overline{\sigma}^{20}_{\mathrm{yy}}$

Table 3. Results for the complete LOK model

$[-\overline{m}_{E_{13}}^-, -\overline{m}_{E_{13}}^+]$	$[-\overline{m}_{E_i}^-, -\overline{m}_{E_i}^+]$	Range [$-\Delta\sigma^{20\,-}_{\mathrm{yy\,mod}}$, $\Delta\sigma^{20\,+}_{\mathrm{yy\,mod}}$]
[-0,10 ; 0,10]	[-0,01 ; 0,01]	$[-2, 3.10^6; 1, 9.10^6] = [-16\%, 14\%]\ \overline{\sigma}^{20}_{\mathrm{yy}}$

A very low LOK level on the stiffness of the elements of the plate (around $\pm 1\%$) leads to nearly the same dispersion of the bounds of the LOK model as a greater LOK level on the amplitude of the force distribution ($\pm 10\%$). This can be explained by the fact that in the case of an isostatic structure a change in the stiffness of the structure does not alter the strain response of the structure significantly.

3.1.5 Reduction of the LOKs

The central question is: how can one reduce the basic LOKs using additional experimental information? The starting state could be an initial overestimated LOK level obtained experimentally or from *a priori* knowledge.

The main principle is that the measured envelope should be included in (but not identical to) the envelope given by the reduced LOK model. This should be verified for any likely situation in "engineering reality".

Let α denote the output of interest, which can be calculated and compared to measured values. α is defined by the operator $\mathbb{L}$ (see Equation (9)). From the initial model, one gets:

$$\Delta\alpha^+_{mod}(\theta) \text{ and } \Delta\alpha^-_{mod}(\theta)$$

while from experiments, one gets:

$$\Delta\alpha^+_{exp}(\theta) \text{ and } \Delta\alpha^-_{exp}(\theta)$$

We have:

$$\Delta\alpha^{+}_{exp}(\theta) \leq \Delta\alpha^{+}_{mod}(\theta) \tag{16a}$$

$$\Delta\alpha^{-}_{exp}(\theta) \leq \Delta\alpha^{-}_{mod}(\theta) \tag{16b}$$

Furthermore, let us assume that for the test being studied α depends essentially on Substructure E^*. Then, the reduction process is limited to the reduction of the basic LOKs related to E^*. One has:

$$\Delta\alpha_{mod} = \mathbb{L}(\mathbb{K}, \underline{f}) - \overline{\alpha} = \Delta\alpha_{E^*} + \sum_{E \in \complement_{E^*}} \Delta\alpha_E \tag{17}$$

Let $m^*(\theta)$ denote the reduced basic LOKs. One should have

$$\Delta\alpha^{+}_{exp}(\theta) \leq \Delta\alpha^{+*}_{mod}(\theta) \tag{18a}$$

$$\Delta\alpha^{-}_{exp}(\theta) \leq \Delta\alpha^{-*}_{mod}(\theta) \tag{18b}$$

, which means that the measured envelope should be included in the envelope given by the reduced LOK model. The main question is: how close to each other are these two envelopes? The answer depends on the LOK on E^* through the test: this LOK is characterized by a coefficient of representativeness $\rho_{E^*} \in [0;1]$ which can be estimated by experience or through calculation if the reasons for the lack of knowledge are known *a priori*. Compared to Equation (18), the inequalities introduced for the reduction process are reversed:

$$\Delta\alpha^{+}_{exp}(\theta) \geq \Delta\alpha^{+*\,\mathrm{worst}}_{mod}(\theta) \tag{19a}$$

$$\Delta\alpha^{-}_{exp}(\theta) \geq \Delta\alpha^{-*\,\mathrm{worst}}_{mod}(\theta) \tag{19b}$$

The right-hand sides are defined as follows:

$$\Delta\alpha^{+*\,\mathrm{worst}}_{mod}(\theta) = \rho_{E^*}\Delta\alpha^{+}_{E^*}(\theta) + \min_{\substack{\mathbb{K}(\theta)\in\mathcal{K}_{m(\theta)} \\ \underline{F}(\theta)\in\mathcal{F}_{m(\theta)}}} \sum_{E \in \complement_{E^*}} [\Delta\alpha_E(\theta)] \tag{20a}$$

$$\Delta\alpha^{-*\,\mathrm{worst}}_{mod}(\theta) = \rho_{E^*}\Delta\alpha^{-}_{E^*}(\theta) - \max_{\substack{\mathbb{K}(\theta)\in\mathcal{K}_{m(\theta)} \\ \underline{F}(\theta)\in\mathcal{F}_{m(\theta)}}} \sum_{E \in \complement_{E^*}} [\Delta\alpha_E(\theta)] \tag{20b}$$

(20c)

3.1.6 Example: reduction of the basic LOKs

Let us examine the application of our method to an actual industrial structure: the Sylda5 satellite support developed by EADS Group, capable of carrying two satellites simultaneously (Figure 2).

3.2 Description of the structure

3.2.1 Experimental data

Free-vibration measurements with 260 sensors were carried out by IABG on behalf of DASA/DORNIER under contract with CNES.

3.2.2 Data for the theoretical model

The model proposed by EADS represents both the support itself and a cylindrical payload which simulates the presence of a satellite resting on the support. This model consists of 38 substructures made of various materials, including orthotropic sandwich materials, aluminum and steel. Since initial measurements had shown that it was absolutely essential to take into account the deformation of the ground under the support, this was modeled very simply using 3 torsional springs, one translational spring and a rigid-body constraint for all the nodes at the junction between the ground and the support. The final model consisted of 27,648 DOFs and 9,728 elements.

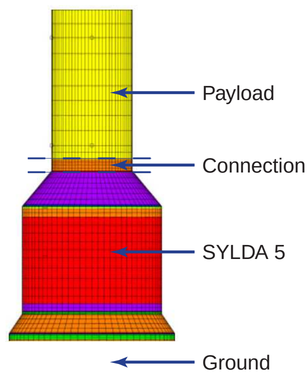

Fig. 7. The Sylda5 model

3.3 Determination of the basic LOKs

The model was first calibrated with the first 12 experimental modes using the method described in Section 3.1.5. The problem was then to determine the remaining LOKs. In order to do that, the structure was divided into 4 main groups of substructures, as described in Figure 7:

- Group $g1$ associated with the cylindrical payload;
- Group $g2$ containing the composite connection between the cylinder and the Sylda5 support;
- Group $g3$ corresponding to the Sylda5 support itself;
- Group $g4$ associated with the ground model.

The reduction process was initiated by setting the initial LOK levels $(\overline{m}_E^{-0}, \overline{m}_E^{+0})$ (and their corresponding laws) *a priori* as shown in Table 4. The experimental information consisted in the eigenmodes and eigenfrequencies measured on the actual structure, which were considered to be the extreme values that would have been obtained if several similar structures had been tested.

Table 4. Initial LOK model

Groups	Law being sought	Range $(\overline{m}_E^{-0}, \overline{m}_E^{+0})$	Relative statistical moments (μ: mean / σ: std. dev.)
E=$g1$	normal	(-0.25,0.25)	($\mu = 0.00$ / $\sigma = 0.097$)
E=$g2$	uniform	(-0.25,0.25)	($\mu = 0.00$ / $\sigma = 0.097$)
E=$g3$	normal	(-0.25,0.25)	($\mu = 0.00$ / $\sigma = 0.097$)
E=$g4$	uniform	(-0.75,0.75)	($\mu = 0.00$ / $\sigma = 0.289$)

Table 5 shows the order in which the reduction was carried out, the data which were used and the results (with $\rho_E = 1$) of the process.

Table 5. Reduced basic LOKs

Group	Experimental data	Reduced basic LOKs		
		Law	$[-\overline{m}_E^-; \overline{m}_E^+]$	Statistical moments
$g3$	$(\Delta\omega^{2\,+}_{4\,\exp}(0.99), \Delta\omega^{2\,-}_{4\,\exp}(0.99))$	normal	$[-0.016; 0]$	$\mu = -0.008$ / $\sigma = 0.003$
$g1$	$(\Delta\omega^{2\,+}_{8\,\exp}(0.99), \Delta\omega^{2\,-}_{8\,\exp}(0.99))$	normal	$[0; 0.144]$	$\mu = 0.072$ / $\sigma = 0.028$
$g4$	$(\Delta\omega^{2\,+}_{6\,\exp}(0.99), \Delta\omega^{2\,-}_{6\,\exp}(0.99))$	uniform	$[0; 0.435]$	$\mu = 0.218$ / $\sigma = 0.126$
$g2$	$(\Delta\omega^{2\,+}_{3\,\exp}(0.99), \Delta\omega^{2\,-}_{3\,\exp}(0.99))$	uniform	$[-0.060; 0]$	$\mu = -0.030$ / $\sigma = 0.012$

These results demonstrated the good quality of the calibrated model of the support ($g3$) and of the model of the connector (both within a few %), but pointed out the oversimplifications in the ground model, resulting in a large LOK, which suggested the use of the specific process described in [3].

The whole reduction process was started again from scratch, using a reduced basis consisting of the first eight modes of the structure and the same experimental data as before. The corresponding results are given in Table 6. The value of the coefficient of representativeness taken here is $\rho_E = 1$ for all substructure E.

Table 6. Reduced basic LOKs taking into account large values of the LOK for the ground (Group $g4$)

Group	Experimental data	Reduced basic LOKs		
		Law	$[-\overline{m}_E^-; \overline{m}_E^+]$	Statistical moments
$g3$	$(\Delta\omega^{2\,+}_{4\,\exp}(0.99), \Delta\omega^{2\,-}_{4\,\exp}(0.99))$	normal	$[-0.016; 0]$	$\mu = -0.008$ / $\sigma = 0.003$
$g1$	$(\Delta\omega^{2\,+}_{8\,\exp}(0.99), \Delta\omega^{2\,-}_{8\,\exp}(0.99))$	normal	$[0; 0.144]$	$\mu = 0.072$ / $\sigma = 0.028$
$g4$	$(\Delta\omega^{2\,+}_{6\,\exp}(0.99), \Delta\omega^{2\,-}_{6\,\exp}(0.99))$	uniform	$[0; 0.521]$	$\mu = 0.261$ / $\sigma = 0.150$
$g2$	$(\Delta\omega^{2\,+}_{3\,\exp}(0.99), \Delta\omega^{2\,-}_{3\,\exp}(0.99))$	uniform	$[-0.060; 0]$	$\mu = -0.030$ / $\sigma = 0.012$

Except for the LOK associated with the ground, the basic LOKs of the groups were unchanged. In order to evaluate the quality of the results of the reduction process, one can calculate the effective LOK for Mode 1 (which was not used) and compare this with the corresponding experimental values from Table 7. One can observe that the constraints for Mode 1 are properly verified, which shows that the results obtained with the other modes are consistent.

Table 7. Comparison of 99%-values for Mode 1

i	$\overline{\omega}_i^2 - \Delta\omega_{\mathrm{i\,mod}}^{2-}$	$\overline{\omega}_i^2 - \Delta\omega_{\mathrm{i\,exp}}^{2-}$	$\overline{\omega}_i^2$	$\overline{\omega}_i^2 + \Delta\omega_{\mathrm{i\,exp}}^{2+}$	$\overline{\omega}_i^2 + \Delta\omega_{\mathrm{i\,mod}}^{2+}$
1	$1.01\,10^3$	$1.02\,10^3$	$1.02\,10^3$	$1.05\,10^3$	$1.06\,10^3$

4 Conclusion

A new modeling approach taking into account all types of uncertainties or error sources has been developed. This technique enables one to evaluate the ”envelope” of the response of the family of actual structures being considered and subjected to an uncertain environment. This LOK theory is also an answer to model validation with respect to engineering reality. Additional developments will focus on specific tools implementing this approach, especially when the basic LOKs are relatively large. The extension to other sources of error besides stiffness and excitation errors is also in progress. Such a modeling scheme is suitable for robust design.

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